The law of reflection:
\[ \theta_1 = \theta_1' \]

The law of refraction:
\[ n_2 \sin \theta_2 = n_1 \sin \theta_1 \] Snell’s Law

Image formation
Chapter 23

Ray Optics - Applications: Image Formation
• Images are always located by extending diverging rays back to a point at which they intersect

• Images are located either at a point from which the rays of light \textit{actually} diverge or at a point from which they \textit{appear} to diverge

• To find the image it is usually enough to find intersection of just two rays!

• Magnification = \frac{\text{image height}}{\text{object height}}
Flat Refracting Surface

\[ n_2 \sin \theta_2 = n_1 \sin \theta_1 \]

Snell’s Law

\[ \sin \theta_2 \approx \theta_2 \approx \frac{d}{q} \]

\[ \sin \theta_1 \approx \theta_1 \approx \frac{d}{p} \]

\[ n_2 \frac{d}{q} = n_1 \frac{d}{p} \]

\[ q = p \frac{n_2}{n_1} \]

Image is always virtual
Flat mirror
Flat Mirror

- One ray starts at point \( P \), travels to \( Q \) and reflects back on itself
- Another ray follows the path \( PR \) and reflects according to the law of reflection
- The triangles \( PQR \) and \( P'QR \) are congruent
- \( h = h' \) - magnification is 1.
Geometric Optics - Applications: Thin Lenses
Thin Lenses

“Thin” means that the width is much smaller than the radius of curvature.

- Biconvex
- Convex–concave (a)
- Plano–convex
- Biconcave
- Convex–concave (b)
- Plano–concave
Thin Lenses

The thin lens is characterized only by one parameter – FOCAL LENGTH.

Thin Lens Equation:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

Object Distance  Image Distance  Focal Length

\( s' = ? \)
Thin Lenses: Focal Length

\[ f = ? \]

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Strategy of Finding \( f \):

- Define positive and negative sides
  - Usually the left side (the side where the object is) is defined as a positive
- You are going from the positive side,
- Then the first lens surface is defined as surface “1” with the radius of curvature \( R_1 \) and the center of curvature at point \( A_1 \)

Sign rule:
- \( R_1 < 0 \) if \( A_1 \) is in the positive “side”
- \( R_1 > 0 \) if \( A_1 \) is in the negative “side”

The same for the second surface:
- \( R_2 < 0 \) if \( A_2 \) is in the positive “side”
- \( R_2 > 0 \) if \( A_2 \) is in the negative “side”
Focal Length: Examples

\[ f > 0 \]

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ R_1 < 0 \]
\[ R_2 > 0 \]
\[ f < 0 \]

\[ R_1 > 0 \]
\[ R_2 > R_1 \]
\[ \frac{1}{R_2} < \frac{1}{R_1} \]

\[ f > 0 \]

\[ |R_1| > |R_2| \]
\[ \frac{1}{|R_1|} < \frac{1}{|R_2|} \]
\[ \frac{1}{f} = (n - 1) \left( \frac{-1}{|R_1|} + \frac{1}{|R_2|} \right) > 0 \]

\[ R_1 < 0 \]
\[ R_2 < 0 \]
Thin Lenses

\[ f > 0 \]

Converging lens

- Biconvex
- Convex-concave
- Plano-convex

(a)

They are thickest in the middle

\[ f < 0 \]

Diverging lens

- Biconcave
- Convex-concave
- Plano-concave

(b)

They are thickest at the edges
Thin Lenses: Sign Conventions for \( s, s' \)

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

Lateral magnification:

\[
M = \frac{h'}{h} = -\frac{s'}{s}
\]

- \( s > 0 \) \( s' < 0 \)
- \( s < 0 \) \( s' > 0 \)

\( h' > 0 \)

(the same rule as for mirrors!)
Thin Lenses: Numerical Strategy

- Find the focal length $f$
- From the Thin Lens Equation find $s'$ ($s$ is known)

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}}$$

- From the sign of $s'$ find the position of image
- Find magnification

$$M = \frac{h'}{h} = -\frac{s'}{s}$$
Thin Lenses: Focal Points
• If $s \gg f$, then

$$\frac{1}{s} \ll \frac{1}{f}$$

and

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = f$$

Because light can travel in either direction through a lens, each lens has two focal points.

However, there is only one focal length.
• If \( s \gg f \), then

\[
\frac{1}{s} \ll \frac{1}{f}
\]

and

\[
s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = f
\]

• \( s' \) is negative
Thin Lenses: Ray Diagram
Converging Lenses

For a converging lens, the following three rays (two is enough) are drawn:

- **Ray 1** is drawn parallel to the principal axis and then passes through the focal point on the back side of the lens.

- **Ray 2** is drawn through the center of the lens and continues in a straight line.

- **Ray 3** is drawn through the focal point on the front of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.
Converging Lenses: Example 1

The image is real
The image is inverted
The image is on the back side of the lens

\[ s' = \frac{1}{\frac{1}{s} - \frac{1}{f}} = \frac{sf}{s-f} > 0 \]

\[ M = \frac{h'}{h} = -\frac{s'}{s} < 0 \]
Converging Lenses: Example 2

\[ f > s > 0 \]

- The image is virtual
- The image is upright
- The image is larger than the object
- The image is on the front side of the lens

\[
s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = \frac{sf}{s - f} < 0
\]

\[
M = \frac{h'}{h} = -\frac{s'}{s} > 0
\]
For a diverging lens, the following three rays (two is enough) are drawn:

- **Ray 1** is drawn parallel to the principal axis and emerges directed away from the focal point on the front side of the lens
- **Ray 2** is drawn through the center of the lens and continues in a straight line
- **Ray 3** is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis
Diverging Lenses: Example

- The image is virtual
- The image is upright
- The image is smaller
- The image is on the front side of the lens

\[ f < 0 \]

\[ s' = \frac{1}{1 - \frac{1}{s}} = \frac{sf}{s - f} < 0 \]

\[ M = -\frac{h'}{h} = -\frac{s'}{s} > 0 \]
Image Summary

• For a **converging lens**, when the object distance is greater than the focal length \( s > f \)
  – The image is real and inverted

• For a **converging lens**, when the object is between the focal point and the lens, \( s < f \)
  – The image is virtual and upright

• For a **diverging lens**, the image is always virtual and upright
  – This is regardless of where the object is placed
Combination of Two Lenses
The image formed by the first lens is located as though the second lens were not present.

The image of the first lens is treated as the object of the second lens.

Then a ray diagram is drawn for the second lens.

The image formed by the second lens is the final image of the system.

If the image formed by the first lens lies on the back side of the second lens, then the image is treated as a virtual object for the second lens.

- $s$ will be negative.

The overall magnification is the product of the magnification of the separate lenses.
Examples

Find the image of the object (the position and magnification) for lenses (1) and (2) if \( p = 10 \text{ m} \) and radii of curvature of the lenses are 5 m and 1 m. The index of refraction is \( n = 1.5 \).

For lens (1):
- \( R_1 = 5 \text{ m} \)
- \( R_2 = -1 \text{ m} \)
- \( n = 1.5 \)

\[
\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
= (1.5-1) \left( \frac{1}{5} + \frac{1}{1} \right) = 0.6 \text{ m}^{-1}

f \approx 1.7 \text{ m}

\[
q = \frac{1}{f} \left( \frac{1}{p} - 1 \right) = pf - \frac{10 \cdot 1.7}{10 - 1.7} \approx 2 \text{ m}
\]

\[
M = -\frac{q}{p} = -\frac{2}{10} = -0.2
\]

For lens (2):
- \( R_1 = -5 \text{ m} \)
- \( R_2 = 1 \text{ m} \)
- \( n = 1.5 \)

\[
\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
= (1.5-1) \left( \frac{1}{5} - \frac{1}{1} \right) = -0.6 \text{ m}^{-1}

f \approx -1.7 \text{ m}

\[
q = \frac{1}{f} \left( \frac{1}{p} - 1 \right) = pf - \frac{-10 \cdot 1.7}{10 + 1.7} \approx -1.45 \text{ m}
\]

\[
M = -\frac{q}{p} = \frac{1.45}{10} = 0.145
\]
\[ f = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ f > 0 \]

\[ f < 0 \]

\[ s > 0 \quad s' < 0 \]
\[ s < 0 \quad s' > 0 \]

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

\[ M = \frac{h'}{h} = -\frac{s'}{s} \]
Resolution
Resolution

- The ability of optical systems to distinguish between closely spaced objects.
- If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished.
  
  The images are said to be resolved.

- If the two sources are close together, the two central maxima overlap and the images are not resolved.
Resolution, Rayleigh’s Criterion

Rayleigh’s criterion:
When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved

Resolution of a slit:
> Since $\lambda \ll a$ in most situations, $\sin \theta$ is very small and $\sin \theta \sim \theta$
> Therefore, the limiting angle (in rad) of resolution for a slit of width $a$ is

$$\theta_{\min} = \theta_{\text{dark}} = \frac{\lambda}{a}$$

> To be resolved, the angle subtended by the two sources must be greater than $\theta_{\min}$
Resolution: Circular Aperture

- The diffraction pattern of a circular aperture consists of a central bright disk surrounded by progressively fainter bright and dark rings.
- The limiting angle of resolution of the circular aperture is
  \[ \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \]
  - \( D \) is the diameter of the aperture.

The images are well resolved.

The images are just resolved.

The images are unresolved.